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Technical Memorandum

OBSERVER'S POSITION AND VELOCITY DETERMINATION USING SATELLITE DOPPLER MEASUREMENTS (U)

by E. W. HIX

THE JOHNS HOPKINS UNIVERSITY ▪ APPLIED PHYSICS LABORATORY

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THE JOHNS HOPKINS UNIVERSITY • APPLIED PHYSICS LABORATORY
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ABSTRACT

Existing navigation techniques using satellite doppler shift measurements are sensitive to observer velocity errors. A model is developed for determining the position and velocity of the observer with the result that the navigation process will be independent of external velocity source error. The position and velocity determination will be feasible provided that precision reference oscillators with instabilities of about 2 parts in 10^{12} be used in the satellite and the navigator's equipment. Relativistic and tropospheric effects must be accounted for in the model. The feasibility also depends on the satellite velocity error being less than 0.05 meter per second.

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I. INTRODUCTION

Existing navigation techniques using the measured doppler shift of a satellite signal with respect to an observer moving on the surface of the earth or at a measured altitude require initial estimates of observer position and velocity. In these techniques the position of the navigator is computed by least squares estimation at some epoch and requires the use of velocity estimates from other sources which are assumed to be correct. This paper demonstrates the feasibility of determining both position and velocity from the satellite doppler shift data.

The doppler shift is measured simultaneously on two coherent, harmonically related frequencies. A continuous real time correction for ionospheric refraction is made to first order [1], [2], [3] and [4]. The satellite borne oscillator also serves as the master clock for the navigation system. A precise navigation program requires that the navigation system clock be synchronized with the satellite clock. System concepts of satellite doppler navigation are given in considerable detail by Guier and Weiffenbach [1], [2], [3] and [4].

Navigation programs in current use determine only two state variables, latitude and longitude, and an additional parameter, frequency bias. The frequency bias, f_γ, is the difference in the frequencies of the satellite oscillator and the receiver reference oscillator. Typical f_γ determinations in a navigation fix range from one part in 10^8 to one part in 10^9. The f_γ terms must be removed, or else be precisely known, during the navigation process or an error will be induced in the latitude. The stability of existing oscillators is such that the f_γ term can be represented as a constant during the time of the satellite pass. That f_γ may be

considered constant implies that the doppler residual due to this term is an even function with respect to the time of satellite closest approach, τ_c. It is shown on Figure 1 that the doppler residual due to a latitude error is also an even function with respect to τ_c. The f_γ terms must be removed or the fitting process will misconstrue the frequency bias as a latitude error.

The major uncorrected error source, when it occurs, in non-stationary navigation is due to the use of inaccurate velocity data in the computing procedure. The effect of input velocity errors in current computational methods is clearly shown on Figures 2, 3, 4 and 5. The magnitude of the position error is dependent on the magnitude of the velocity error and on the relative geometry of the navigator observing the satellite. The sign of the position error is dependent on observer-satellite geometry and the direction of satellite motion relative to the observer.

Figures 2 and 3 show, respectively, the magnitude of longitude error, $\delta\lambda$ in nautical miles, due to an input velocity north error, δV_N in knots, and the second order latitude error, $\delta\varphi$ in nautical miles, due to δV_N. The abscissa of Figures 2, 3, 4 and 5 is given in terms of the elevation angle at time of satellite closest approach, measured with respect to the topocentric horizon. Figures 4 and 5 show the magnitude of latitude error, $\delta\varphi$, due to an input velocity east error, δV_E in knots, and the magnitude of the second order longitude error, $\delta\lambda$, due to δV_E. The position error curves due to velocity error represented by Figures 2, 3, 4 and 5 were obtained in a computer simulation and have been verified by analytic methods. The position errors due to velocity errors are linear over a large range, about 20 knots, of velocity errors. The modeling necessary to determine position and velocity from the satellite doppler data will now be outlined.

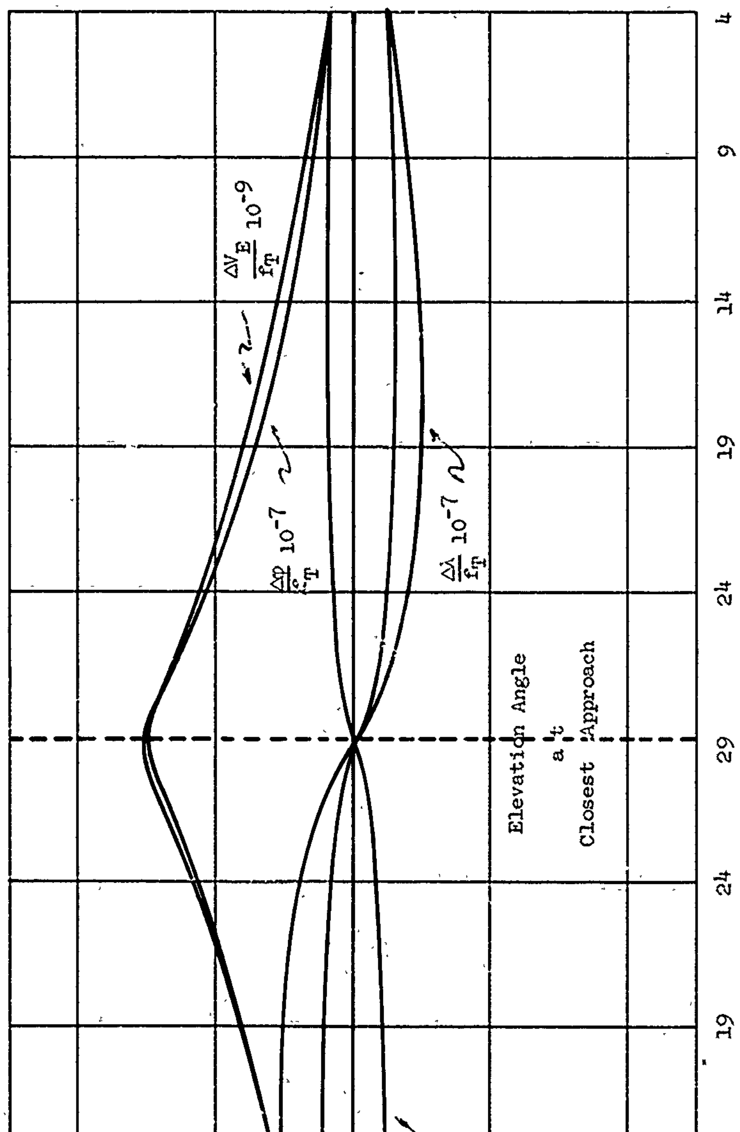

Fig. 1 Residuals Referred to Transmitted Frequency

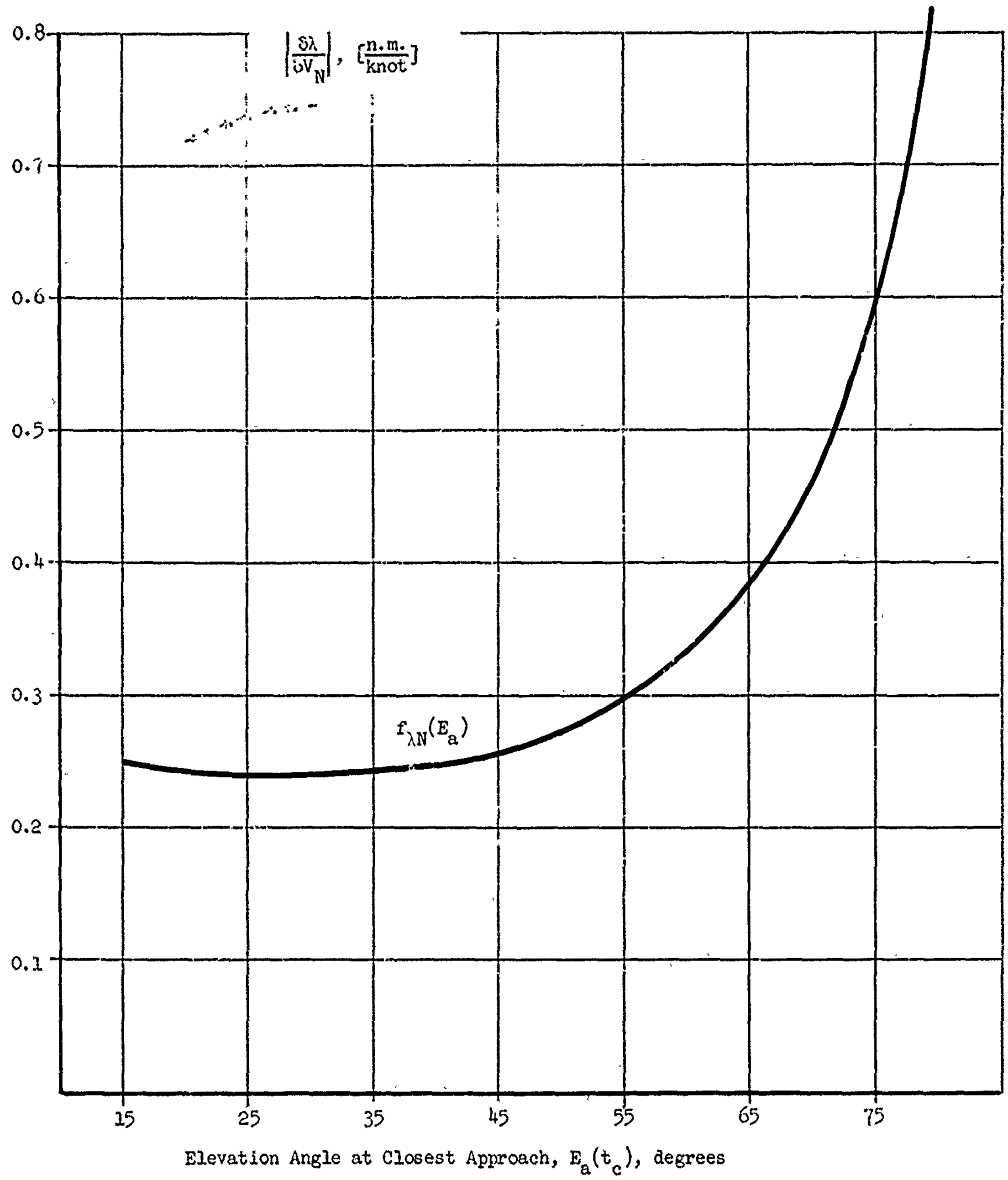

Fig. 2 Longitude Error per Velocity North Error

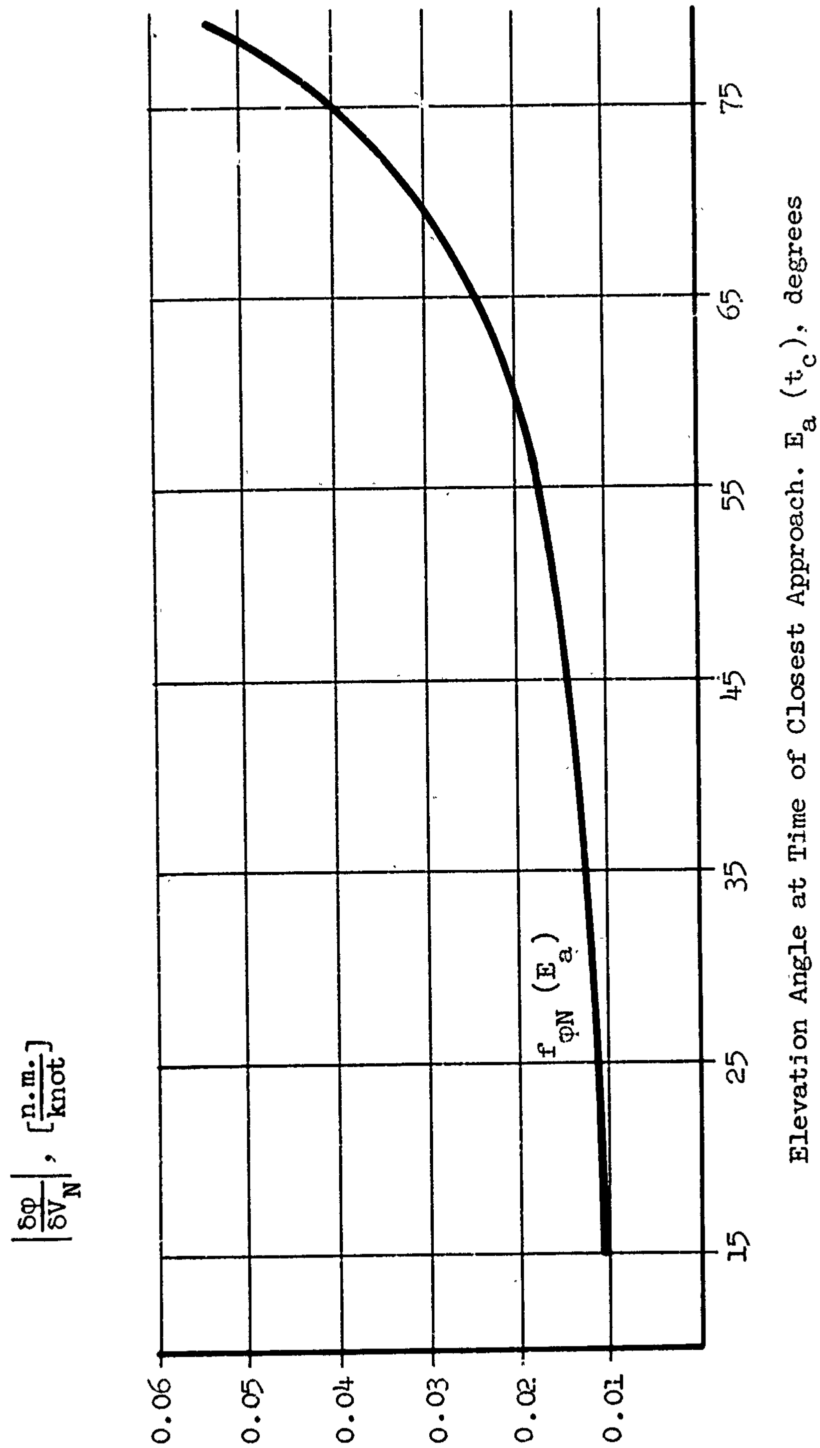

Fig. 3 Latitude Error per Velocity North Error

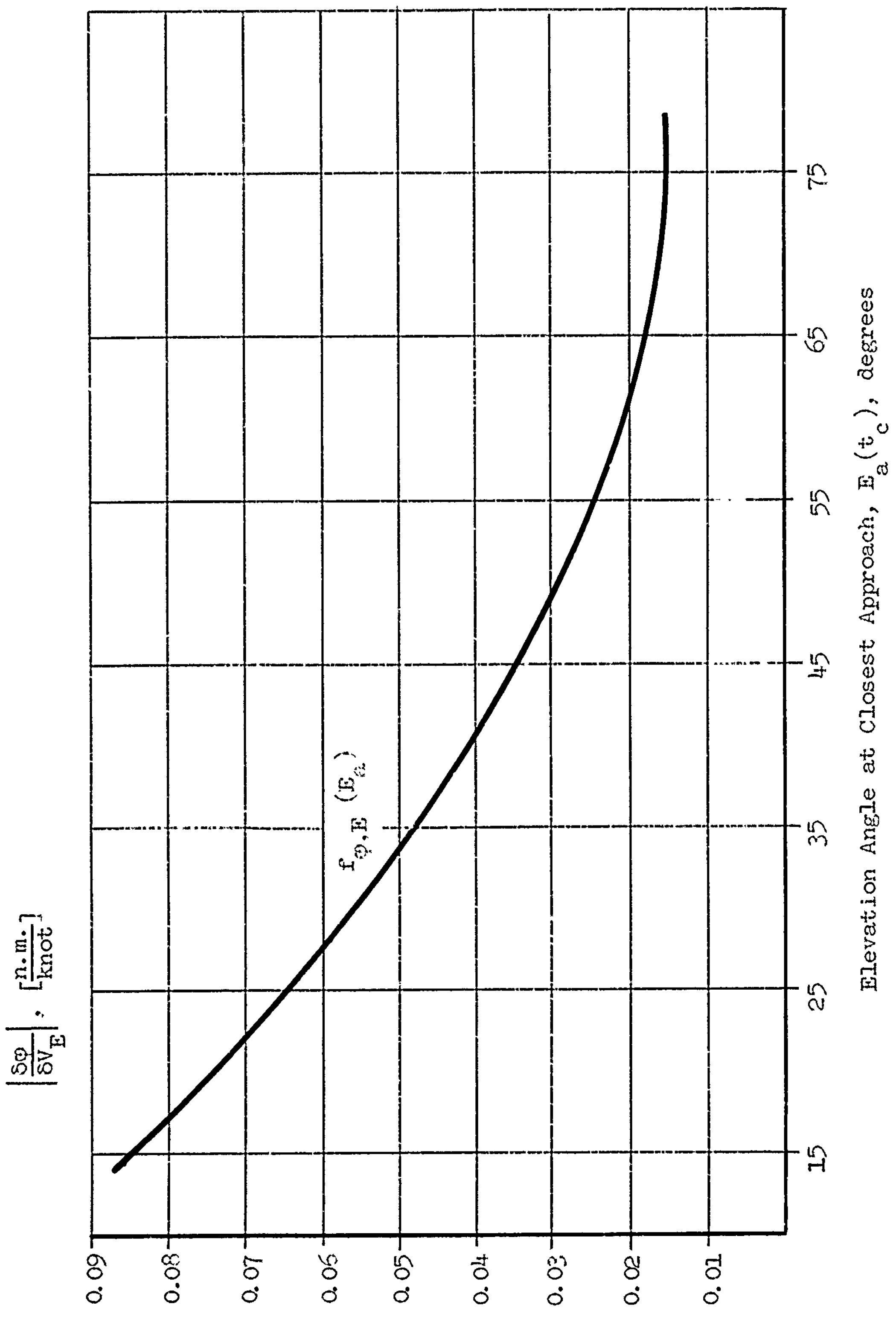

Fig. 4 Latitude Error per Velocity East Error

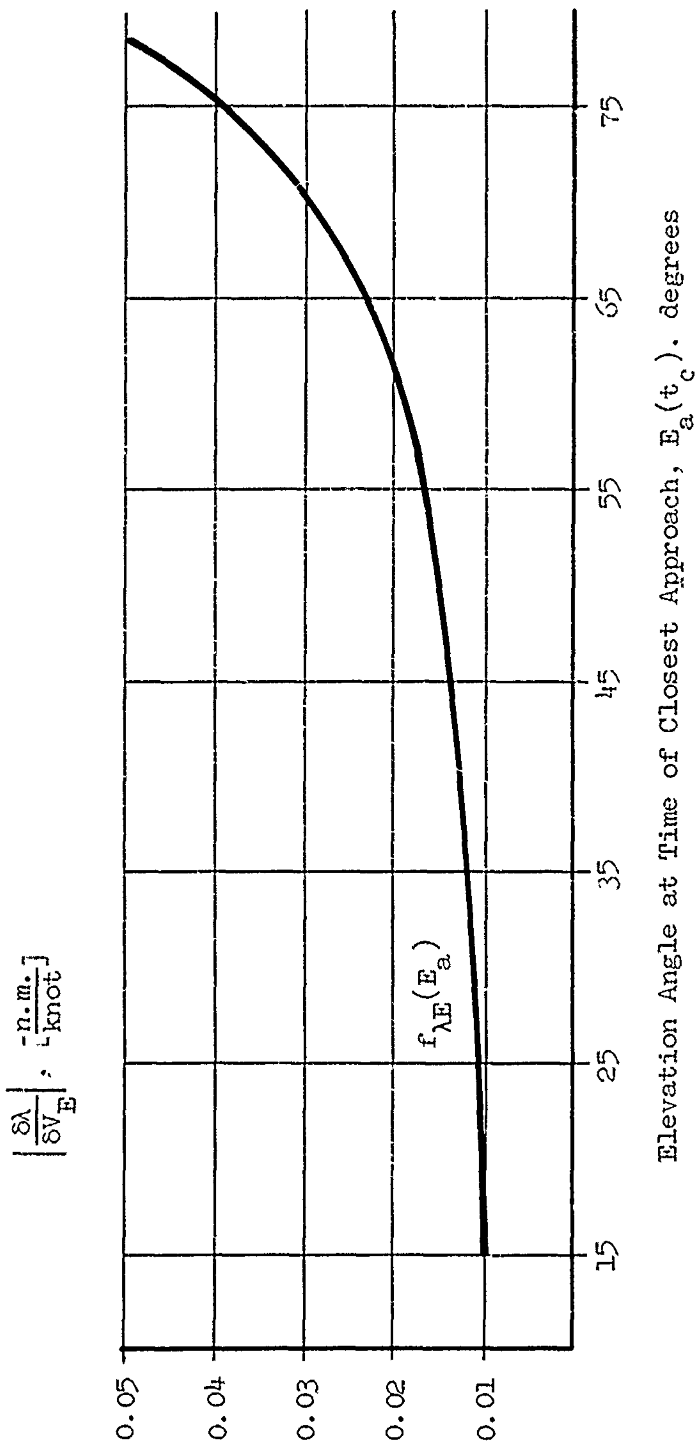

Fig. 5 Longitude Error per Velocity East Error

II. DISCUSSION

The expression for the doppler shift of the satellite signal as seen by the observer, including general relativistic effects [5], [6], [7], thru order c^2 is given by (1)

$$(1)\ \Delta f(t,\tau) = -\frac{f_T}{c}\left\{\hat{\rho}(t,\tau)\cdot\dot{\bar{\rho}}(t,\tau) + \frac{1}{c}\left[(\hat{\rho}(t,\tau)\cdot\dot{\bar{R}}_s(t))^2 + (\hat{\rho}(t,\tau)\cdot\dot{\bar{R}}_s(t))(\hat{\rho}(t,\tau)\cdot\dot{\bar{r}}_N(\tau)) - \frac{1}{2}(\dot{R}_s^2(t) - \dot{r}_N^2(\tau)) + \frac{GM_E}{R_s(t)r_N(\tau)}(R_s(t) - r_N(\tau))\right]\right\}$$

where
$t \equiv$ satellite clock time
$\tau \equiv$ observer's clock time
$\bar{R}_s \equiv$ satellite position
$\bar{r}_N \equiv$ observer's position
$\bar{\rho}(t,\tau) \equiv \bar{R}_s(t) - \bar{r}_N(\tau)$
$\hat{\rho}(t,\tau) \equiv \bar{\rho}(t,\tau)/|\bar{\rho}(t,\tau)|$
$c \equiv$ velocity of light
$G \equiv$ Newtonian gravitation constant
$M_E \equiv$ earth mass
$f_T \equiv$ source frequency
$\Delta f \equiv$ doppler frequency
$\tau = t + |\bar{\rho}(t,\tau)|/c$

and the dot $(\cdot)$ indicates time derivative.

Equation (1) is the modeled, or theoretical, doppler frequency to be computed in the navigation process. Relativistic corrections are not necessary for latitude and longitude position determinations. The relativistic terms must be included for position and velocity fixes, since velocity determination from doppler data requires that the frequency be relatively bias free, as will be shown later. Examination of the terms of order c^2 in (1) reveals that these terms contribute a frequency bias

term of the order of one part in 10^9 for an orbit with a semi-major axis of about 1.16 earth radii. The maximum doppler shift relative to the source frequency is about 2 parts in 10^5.

The first order approximation of (1) is

$$\Delta f(\tau) = -\frac{f_T}{c}\hat{\rho}\cdot(\dot{\bar{R}}_s - \dot{\bar{r}}_N) \tag{2}$$

It is clear from (2) that errors in the observer's velocity and position contribute to errors in the doppler frequency. It is also evident from (2) that the satellite ephemeris, $\bar{R}_s$ and $\dot{\bar{R}}_s$ is used as the reference in the determination of the doppler frequency. Any errors in the ephemeris will contribute errors to the observer's position and velocity determination.

The least squares adjustment of the observer's position and velocity to measured doppler points μ and other apriori data is formulated by (3).

$$F = \frac{1}{N}\sum_{\mu=1}^{N}\left[\Delta f(t_\mu, \tau_\mu|\bar{x}) - \Delta f_v(\tau_\mu)\right]^2 \tag{3}$$

where $\Delta f(t_\mu, \tau_\mu|\bar{x})$ represents the theoretical doppler data, $\Delta f_v(\tau_\mu)$ the measured doppler data and $\bar{x}$ the state vectors to be minimized. The components of the state vector are

$x_1 = \varphi$, observer's latitude

$x_2 = \lambda$, observer's longitude

$x_3 = \dot{\varphi}$, observer's angular velocity along a meridian

$x_4 = \dot{\lambda}$, observer's angular velocity along a circle of latitude.

The remaining components of the state vector r_N and $\dot{r}_N$ will be constraints on the above formulation; details for computation will be given later.

Expressing the theoretical doppler shift by a Taylor series to linear terms we have

$$(4) \qquad \Delta f(t_\mu, \tau_\mu|\bar{x}) = \Delta f(t_\mu, \tau_\mu|\bar{x}_e) + \sum_{i=1}^{4} \frac{\partial \Delta f(t_\mu, \tau_\mu|\bar{x}_e)}{\partial x_i} \Delta x_i$$

where the subscript e represents the initial estimates. Define, for convenience

$$b_i \equiv \frac{\partial \Delta f(t_\mu, \tau_\mu|\bar{x}_e)}{\partial x_i}$$

and

$$R \equiv \Delta f(t_\mu, \tau_\mu|x_e) - \Delta f_v(\tau_\mu), \text{ the doppler residuals.}$$

The least squares adjustment may now be written as

$$(5) \qquad F = \frac{1}{N} \sum_{\mu=1}^{N} \left[R + \sum_{i=1}^{4} b_i \, \Delta x_i \right]^2$$

The usual minimization of F by $\frac{\partial F}{\partial \Delta x_\ell} = 0, \quad \ell = 1, 2, 3, 4$ results in the normal equations (6).

$$(6) \qquad \sum_{\mu=1}^{N} R b_\ell + \sum_{\mu=1}^{N} \sum_{i=1}^{4} b_i \, b_\ell \, \Delta x_i = 0, \qquad \ell = 1, 2, 3, 4$$

Equation (6) will be solved for Δx_i. An improved estimate of x_i and thereby an improved estimate of the theoretical doppler value can be computed. The process is iterated until the magnitudes of the Δx_i are sufficiently small such that the corrected values of x_i are said to represent the navigation solution.

The normal equations (6) are constrained to yield a solution on the surface of the geoid in the case of ship navigation or on the surface of a co-geoid in the case of aircraft navigation. The constraint is applied by means of the theoretical doppler evaluation on the geoid or co-geoid while the partial derivatives allow only movement of the observer's position and velocity on the geoid or co-geoid for ship and aircraft navigation respectively. The explicit constraint of the state vector $\overline{r}_N$ is given by equation (8).

The general form of the partial derivatives needed for the normal equations is given by (7).

$$(7) \qquad b_i = \frac{f_T}{c}\left\{\hat{\rho} \cdot \frac{\partial \overline{r}_N}{\partial x_i}\right\}, \qquad i = 1, 2, 3, 4$$

The partial derivatives may be obtained in a straightforward manner beginning with equations (2), (8) and (11).

The navigator's position on the geoid in earth fixed cylindrical coordinates, is

$$(8) \qquad \begin{aligned} \zeta_c &= R_o[1 + (1 - f)^2 \tan^2 \varphi(\tau)]^{-1/2} + (h + H) \cos \varphi(\tau) \\ \lambda_c &= \lambda \\ z_c &= \zeta_c(1 - f)^2 \tan \varphi(\tau) + (h + H) \sin \varphi(\tau) \end{aligned}$$

where R_o and f are the equatorial radius and flattening of the ellipsoid respectively, h is the altitude of the antenna with respect to the geoid, H is the altitude of the geoid with respect to the reference ellipsoid and is usually defined as the geoidal height. Geoidal height may be computed

to a sufficiently accurate approximation as the difference between the geoid potential function U, and the reference ellipsoid potential function V, divided by the local gravity, here taken as the partial derivative of V with respect to r_N.

$$(9)\quad H = \frac{R_o\left\{C_3^0 P_3^0 \sum_{n=5}^{\infty} C_n^0 P_n^0 + \sum_{n=2}^{N_{max}} \sum_{m=1}^{n} P_n^m [C_n^m \cos m\lambda + S_n^m \sin m\lambda]\right\}}{1 + 3C_2^0 P_2^0 + 5C_4^0 P_4^0 - \frac{(\omega_e R_o \cos\varphi)^2}{\mu}}$$

where P_n^m are spherical harmonics; C_n^m, S_n^m are the coefficients of the harmonics and μ is the product of the Newtonian gravitation constant g, and the mass of the earth, M. We use the geopotential function given by Guier and Newton, [8]. The usual definitions of φ and λ used in evaluating H are geocentric latitude and longitude, respectively. Negligible errors are obtained by using geodetic latitude and longitude. Geoidal height is a well behaved function, changing slowly with respect to latitude and longitude increments, and for this reason it will only be necessary to compute H once per navigation fix provided the original estimates of φ and λ are correct to within 2 degrees.

The altitude of the antenna with respect to the mean sea level, the geoid in the case of a ship, is treated as a measured system constant. This completes the specification of the constraints $\bar{r}_N$ and $\dot{\bar{r}}_N$ for the case of ship navigation. Aircraft navigation requires external measurement of h to give the estimate of the state vector $\bar{r}_N$. Sufficiently fast sampling rates of h will allow an estimate of $\dot{\bar{r}}_N$ to be given.

The effect of altitude, h or H, errors in the computation of the navigator's constrained radius is shown on Figure 6. The residuals of this

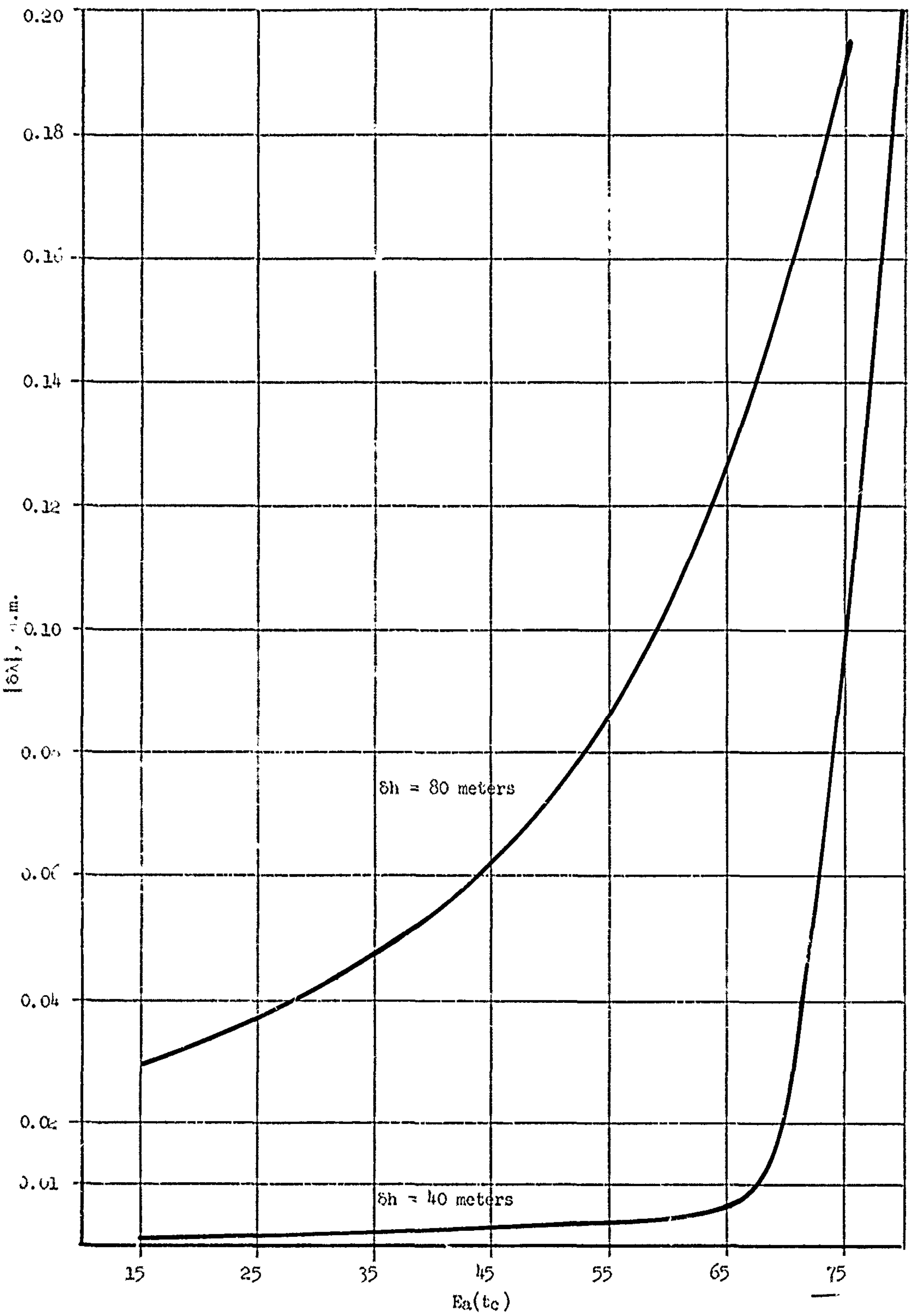

Fig. 6 Longitude Error, δλ due to Antenna Height Error, δh

effect were given on Figure 1 and shown to be an odd function. Again, the effect of the odd function residual can be misinterpreted in the least squares adjustment as either a longitude error or a velocity north error.

Figure 6 shows the longitude error due to height errors of 40 meters and 80 meters. The relative shape of the curves shows this effect to be non-linear. Generally, this effect is negligible for ship navigation since the capabilities exist to compute the navigator's radius to an accuracy considerably better than 40 meters. Aircraft navigation will require careful measurements and modeling of the least squares procedure. The effect of altitude error on the velocity north is negligible, amounting to a maximum value of about 0.05 knots for an altitude error of 60 meters.

The observer's inertial longitude is given by (10)

$$(10) \qquad \Lambda(\lambda,\tau) = \lambda(\tau) + \Lambda_G(t_o) + \omega_e(\tau - t_o)$$

where $\Lambda_G(t_o)$ is defined as the right ascension of Greenwich meridian at an epoch, here t_o, zero hours UT, and ω_e, the earth's average sidereal rate.

In order to obtain the partial derivatives, we decompose equation (8) into inertial cartesian coordinates using (10) and differentiate as necessary. The partial derivatives needed will become clear if the first order expression for the doppler shift is expanded as

$$\Delta f(t,\tau) = - \frac{f_T}{c} [x\dot{x} + y\dot{y} + z\dot{z}]\rho^{-1}$$

and the general partial derivative is then

$$b_i = \frac{f_T}{c} \{[x \frac{\partial \dot{x}_N}{\partial x_i} + y \frac{\partial \dot{y}_N}{\partial x_i} + z \frac{\partial \dot{z}_N}{\partial x_i} + \dot{x} \frac{\partial x_N}{\partial x_i} + \dot{y} \frac{\partial y_N}{\partial x_i} + \dot{z} \frac{\partial z_N}{\partial x_i}]\rho^{-1}$$

$$- \frac{\dot{\rho}}{\rho^2} [x \frac{\partial x_N}{\partial x_i} + y \frac{\partial y_N}{\partial x_i} + z \frac{\partial z_N}{\partial x_i}]\}, \quad i = 1, 2, 3, 4$$

The least squares adjustment of the data is normally referenced to an epoch since in the general case the latitude, longitude, and velocity components are functions of time, τ. The epoch chosen is τ_c, the observer's time of satellite closest approach.

$$\varphi(\tau) = \varphi(\tau_c) + \int_{\tau_c}^{\tau} \dot{\varphi}(t)dt$$

(12)

$$\lambda(\tau) = \lambda(\tau_c) + \int_{\tau_c}^{\tau} \dot{\lambda}(t)dt$$

Using numerical integration of oblate earth velocities in (12) permits any reasonable maneuver of a ship or aircraft, depending upon the sample rate of the data. The maneuvering capability does require some form of an external dead reckoning device such as an inertial system. In the event no maneuver capability is required during the time of satellite pass, then dead reckoning devices are not necessary for furnishing initial parameter estimates.

Equation (6) represents a positive definite quadratic surface, as is evident from the results given later. The quadratic surface is highly elliptical with the semi-major axes in the direction of longitude and

velocity east with the minimum of the surface at the true zero, that is where errors of latitude, longitude, velocity north and velocity east are zero. Equation (6) is positive definite when the doppler data used in the least squares adjustment covers a sufficient time, or geometrical span. Figure 1 shows the latitude and velocity east residuals to be virtually the same except near the horizon. Use of doppler data near the horizon introduces another systematic error due to tropospheric refraction.

Tropospheric refraction effects can contribute an error which, in general, is an odd function with respect to time of closest approach, [9], [10]. Errors behaving as odd functions will be misinterpreted in the least squares adjustment as false longitude or velocity north residuals, resulting in errors to these determinations. Ordinarily, tropospheric refraction effects need not be accounted for if the portions of the doppler curve between the horizon and approximately ten degrees elevation are deleted from the least squares computation. However, in determination of position and velocity, it has been found necessary to use the doppler curve at elevation angles of about five degrees. Figure 1 shows the doppler residuals, that is the difference between the theoretical, or modeled, doppler data and the measured doppler data for latitude, longitude, velocity north, and velocity east errors. It is clear from Figure 1 that the longitude and velocity north residuals can be identified as odd functions with respect to satellite closest approach while latitude and velocity east residuals are even functions.

The velocity error effects have furnished the motivation in seeking to determine observer's velocity and position from satellite doppler data. Until recently, only partial success in velocity determination had

been achieved. In 1966 doppler data were obtained on a ship maneuvering within range of a shore based theodolite triangulation net. The theodolite data were reduced to give the ship's true position as a function of time. The same doppler data were navigated in two different fitting routines as

a) Determination of frequency, latitude, and longitude.

b) Determination of frequency, latitude, longitude, and velocity north.

The results of the two methods of navigating are shown on Figure 7. The f_γ, φ and λ determination used velocity components from an inertial navigation system. The f_γ, φ, λ and V_N determination used the velocity north component from the inertial system only as an initial estimate in the least squares adjustment. The resultant improvements in the f_γ, φ, λ, V_N navigation fixes are due to the determination of a more accurate velocity north component in the fitting process using satellite doppler data.

Recent experimentation by G. C. Weiffenbach and the author indicates that velocity east may also be determined provided that the satellite oscillator and receiver reference oscillator are sufficiently stable such that the frequency term does not have to be included in the normal equations as indicated by equations (3) thru (6). Weiffenbach and the author have used real world data in arriving at the required oscillator stability. The results from real data and the simulation are in substantial agreement.

The experiment using the simulation program consisted of implementing equation (6) and the constraints noted along with controlled errors representing oscillator instabilities. Figure 8 shows the results of the simulation experiment, that is, the effect of velocity east errors due to oscillator instabilities. The results shown on Figure 6 can be expressed as

$$|\delta V_E| = |\delta f_\gamma| \; g_\gamma(E_a(t_c))$$

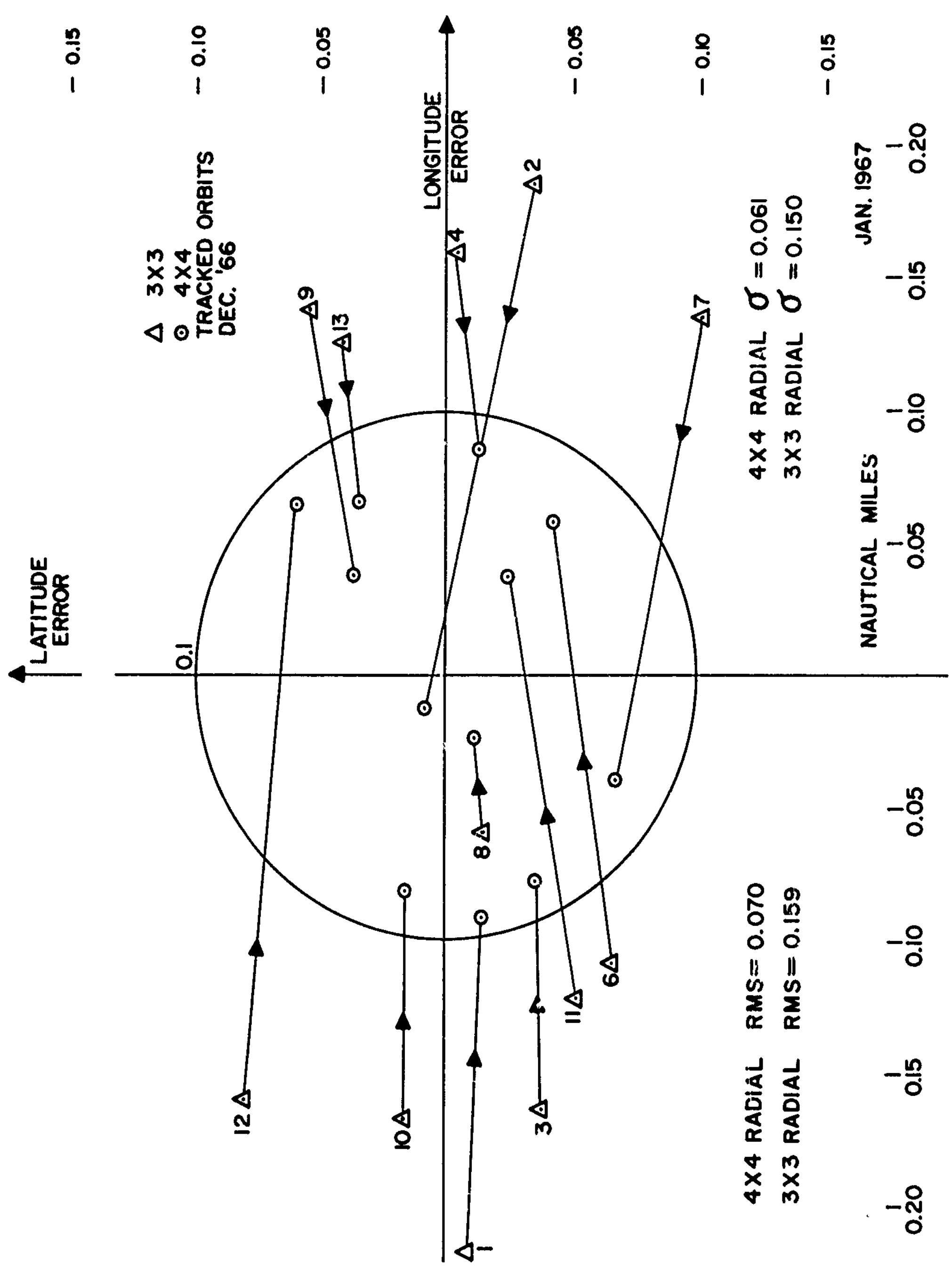

Fig. 7 Effect of Reducing Observer's Velocity Errors

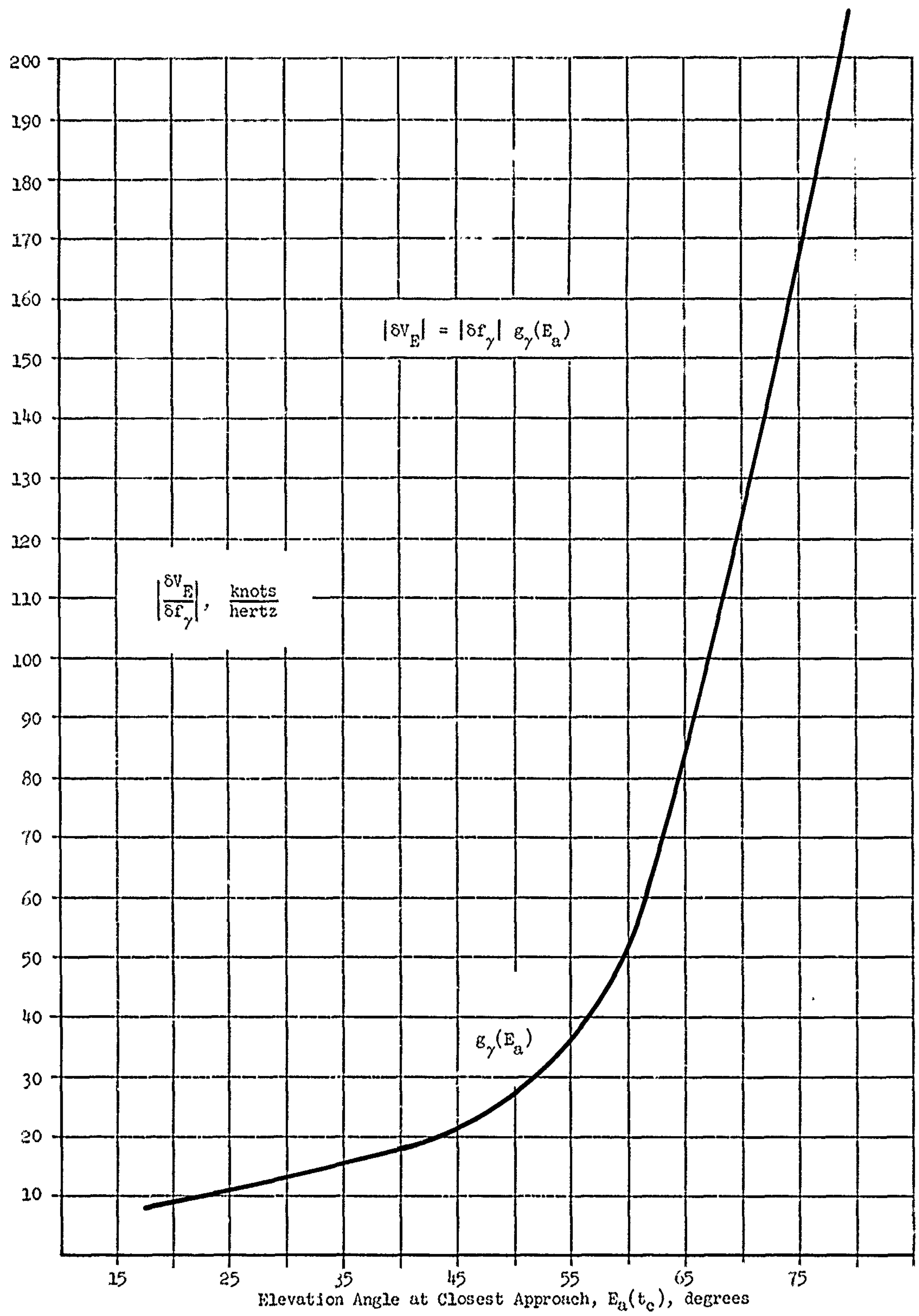

Fig. 8 Velocity East Error per Frequency Bias Error

A value for the relative oscillator stability can be determined by assigning a prescribed value to the allowable velocity east error. It is more meaningful to include the effect of δV_E on the latitude error using the curve of Figure 4 as

$$|\delta\varphi| = |\delta V_E| f_E(E_a(t_c))$$

Substituting,

$$|\delta\varphi| = |\delta f_\gamma| g_\gamma(E_a(t_c)) f_E(E_a(t_c))$$

a value of $|\delta f_\gamma|/f_T$, selected to allow 0.01 nautical mile latitude error, represents a total oscillator instability of 5 parts in 10^{12}. This total oscillator instability includes the combined effects of satellite oscillator and receiver reference oscillator instabilities.

Similar results have been obtained with real data in the experiment by Weiffenbach and the author using navigation results from two separate fixed sites. One fixed site is equipped with a Cesium frequency standard while the other uses a Rubidium reference. At the time of this writing, the Rubidium standard appears superior in short term stability over a 20 minute span.

Assuming that it is possible to acquire atomic frequency standards with a stability of about 2 parts in 10^{12} for both the satellite and receiver reference oscillators then it is probable that velocity east can be determined in the real world with good accuracy. Determination of velocity north has already been demonstrated on an experimental basis, however it must be noted that acquisition of a value for velocity north places rather stringent requirements on satellite velocity. Equation (2)

clearly states that the ephemeris is the navigation reference. Satellite velocity must be accurate to less than 0.05 meters per second if a 0.1 knot determination of velocity north is desired since one knot is 0.514 meters per second.

III. CONCLUSION

Position and velocity have been determined in a computer simulation. The simulation program accounts for all modeling except atmospheric, solar, and magnetic activity. Excluded are ionospheric refraction [1], [3], tropospheric refraction [9],[10], and the effects of drag forces which must be accounted for in the real world. Assuming sufficient oscillator stability and a sufficiently accurate ephemeris, then the accuracy of the doppler satellite navigation system may be theoretically indicated by the computer simulation results given on Figure 9.

Assumptions:

Orbit Velocity Error < 0.05 meters/sec

Oscillator Stability $\approx 2(10)^{-12}$

Conditions:

Observer Velocity $15\sqrt{2}$ kts

Observer at Equator

Observer Input Errors:

Position $6\sqrt{2}$ miles

Velocity $\sqrt{2}$ knots

System Noise 0.01 hertz

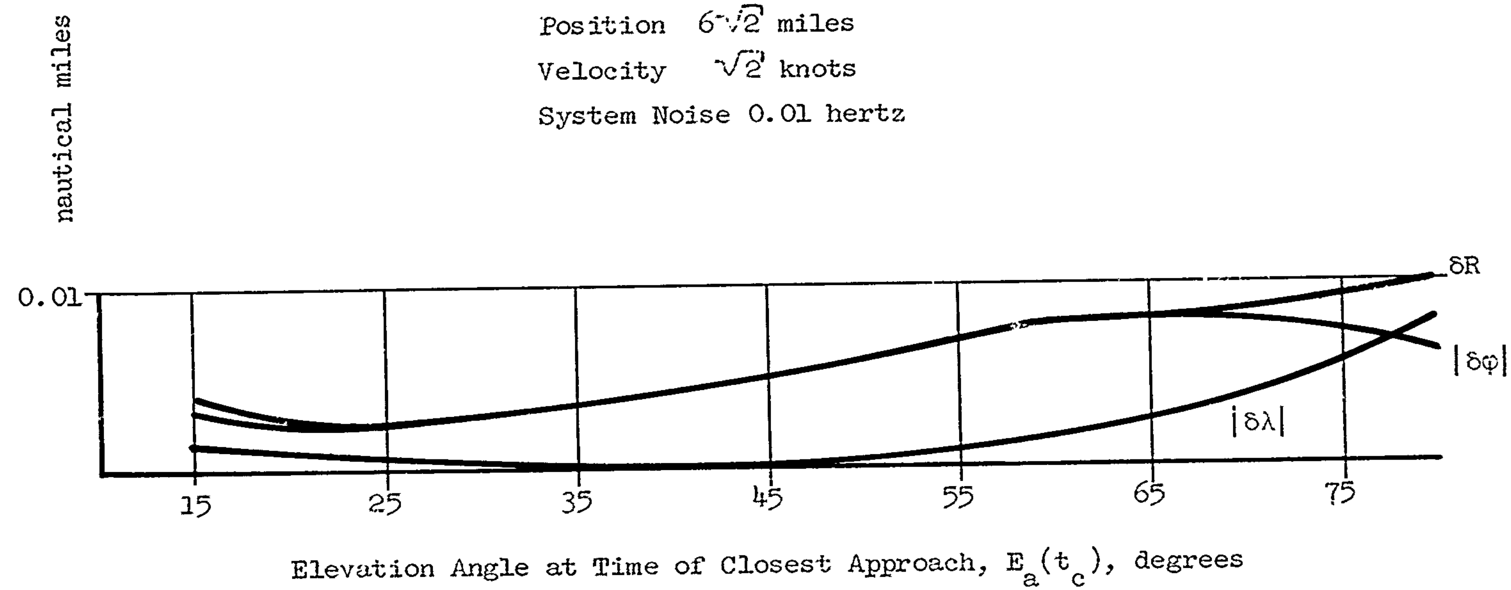

Fig. 9 Theoretical Navigation Accuracy

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13. ABSTRACT

Existing navigation techniques using satellite doppler shift measurements are sensitive to observer velocity errors. A model is developed for determining the position and velocity of the observer with the result that the navigation process will be independent of external velocity source error. The position and velocity determination will be feasible provided that precision reference oscillators with instabilities of about 2 parts in 10^{12} be used in the satellite and the navigator's equipment. Relativistic and tropospheric effects must be accounted for in the model. The feasibility also depends on the satellite velocity error being less than 0. 05 meter per second (U).

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